

Fast Implementation of Color Constancy Algorithms

Jean-Michel Morel^a, Ana Belen Petro^b and Catalina Sbert^b

^a CMLA ENS-Cachan, 61 av. du President Wilson 94235 Cachan cedex, Paris, France;

^bUniv. of Balearic Islands, Ctra. de Valldemossa Km 7.5, 07122 Palma de Mallorca, Spain

ABSTRACT

Color constancy is a feature of the human color perception system which ensures that the perceived color of objects remains relatively constant under varying illumination conditions, and therefore closer to the physical reflectance. This perceptual effect, discovered by Helmholtz, was formalized by Land and McCann in 1971, who formulated the Retinex theory. Several theories have ever since been developed, known as Retinex or color constancy algorithms. In particular an important historic variant was proposed by Horn in 1974 and another by Blake in 1985. These algorithms modify the RGB values at each pixel in an attempt to give an estimate of the physical color. Land's original algorithm is both complex and not fully specified. It computes at each pixel a stochastic integral on an unspecified set of paths on the image. For this reason, Land's algorithm has received many recent interpretations and implementations that attempt to tune down the excessive complexity. In this paper, a fast and exact FFT implementation of Land's, Horn and Blake theories is described. It permits for the first time a rigorous comparison of these algorithms. A slight variant of these three algorithms will be proposed, that makes them into contrast enhancing algorithms. Several comparative experiments on color images illustrate the superiority of Land's model to manipulate image contrast.

Keywords: Retinex, color correction, color constancy, FFT

1. INTRODUCTION

Color constancy is the ability to determine the colors of objects irrespective of the illumination conditions¹ and of the nearby objects color. This is an important characteristic of the Human Visual System (HVS). The HVS is able to compute some descriptors which define the object color independently of the present illumination in the scene and independently of the surrounding objects color. The goal of color constancy research is to achieve these descriptors, which means discounting the effect of illumination and obtaining a canonical color appearance.

Many color constancy theories have been proposed.²⁻⁶ Probably the first is the Land and McCann Retinex theory.⁷⁻⁹ This theory is based on psychophysical experiments using Mondrian patterns. These experiments supported the existence of a quantity named "lightness" which was associated to the objects of the scene regardless of changes in the illumination or in the position of the objects in the scene. These experiments show that the lightness information is processed independently in the three sets of color receptors. Retinex is the first computational model to explain and achieve color constancy. In the past thirty years, numerous Retinex implementations have been published and effort has been made to improve the original Retinex algorithm.¹⁰⁻²⁰

In²¹ the original Retinex algorithm proposed by Land has been rigorously reformulated as a discrete partial differential equation which can be solved using the Fast Fourier Transform. Compared to other Retinex implementations, this new formalization is exact and reduces the computational cost and the number of parameters.

Retinex is in principle designed only to demonstrate a color perception theory. However, most authors modify it to define efficient color correction and image enhancement algorithms.²²⁻²⁶ Following their path, our main aim here is to discuss the slight modifications of the original Retinex yielding efficient image contrast enhancement.

The paper is organized as follows. The next section presents the original Retinex algorithm and a brief overview of several types of Retinex implementations. Using the mathematical formalization of the original

JMM: morel@cmla.ens-cachan.fr

ABP: anabelen.petro@uib.es

CS: catalina.sbert@uib.es

Retinex²¹ Section 3 describes the Fast Fourier Transform implementation. Section 4 displays some results of the proposed algorithm, which is in continuation adapted to gamma-rectified images, the aim being color contrast enhancement. Section 5 exposes some conclusions.

2. ORIGINAL RETINEX ALGORITHM

The basic Retinex model is based on the assumption that the HVS operates with three retinal-cortical systems, each one processing independently the low, middle and high frequencies of the visible electromagnetic spectrum. Each system produces one lightness value which determines, by superposition, the perception of color in the HVS. On digital RGB images, the lightness is represented by the triplet (L_R, L_G, L_B) of lightness values in the three chromatic channels.

Land and McCann observed that edges are the main source of information to achieve color constancy.²⁷ Moreover, they realized that the procedure of taking the ratio between two adjacent points can both detect an edge and eliminate the effect of nonuniform illumination. In an early version²⁸ the lightness information is estimated by computing sequential ratios between values at adjacent points of a series of random paths in the image. Changes above a certain threshold are considered as changes in reflectance. If instead color changes are smaller than the threshold they are considered as illumination changes, and the current ratio is set to one. After computing on many paths, the result on each path is averaged to obtain the lightness. This computation is repeated over all three colors channels and the resulting triplets of lightness correspond to the perceived color. In a posterior work⁸ Land computes the lightness values from the logarithmic values of the sensor response, so that differences are computed instead of ratios.

The formulas describing the lightness L of a pixel $x = (i, j)$ computed by Retinex in each chromatic channel are given in.⁸ The main idea is to consider the spatial chromatic information around each pixel x to compute its lightness. The image data $I(x)$ is the intensity value for each chromatic channel at x . Land and McCann consider a collection of N paths $\gamma_1, \dots, \gamma_N$ starting at j_k and ending at x . Let n_k be the number of pixels of the path γ_k and denote by $x_{t_k} = \gamma_k(t_k)$ for $t_k = 1, \dots, n_k$ and by $x_{t_k+1} = \gamma_k(t_k + 1)$ the subsequent pixel of the path.

Definition 1. The lightness value $L(x)$ of a pixel x in a given chromatic channel is the average of the relative lightness at x over all paths, that is

$$L(x) = \frac{\sum_{k=1}^N L(x; j_k)}{N}, \quad (1)$$

where $L(x; j_k)$ denotes the relative lightness of a pixel x with respect to j_k defined by

$$L(x; j_k) = \sum_{t_k=1}^{n_k} \delta \left[\log \frac{I(x_{t_k+1})}{I(x_{t_k})} \right], \quad (2)$$

and, for a fixed threshold t ,

$$\delta(s) = \begin{cases} s & \text{if } |s| > t \\ 0 & \text{if } |s| < t. \end{cases} \quad (3)$$

2.1 Previous Retinex Implementations

The main issue of Land's theory is its computational complexity, which has led to many attempts at acceleration and simplification, and therefore to many variants. Land and McCann's²⁷ first version used a few piecewise linear paths. Provenzi et al.¹⁹ presented a detailed mathematical analysis of Land's Retinex algorithm concluding with an analytical formula. Based on this mathematical study Marini and Rizzi¹⁸ substituted Brownian paths to the piecewise linear paths. In a recent paper Provenzi et al.²⁰ eventually replaced the paths by more local "random sprays". The problem with these Retinex path-based methods remains their relatively high computational complexity and the many implementation parameters such as the number of paths (or sprays), and their lengths. The final result may depend highly on these parameters.

An iterative version of Retinex developed by Frankle and McCann^{13,14,29} extended the path version. Their algorithms compute long-distance iterations between pixels first, and then progressively move to short-distance interactions. The drawback of these implementations is that the number of iterations is not defined and can strongly influence the final result (see³⁰ for a discussion of this point).

Another variant is the center/surround technique first proposed by Land.³¹ His algorithm computes a distance weighted average and subtracts this average from the intensity value at that point. This technique was then improved by Rahman et al.,¹⁷ who used a Gaussian to compute the blurred image and perform color correction in a single scale. In a later paper this algorithm was extended to a multiscale Retinex.³² The algorithms in the center/surround class are faster than the path-based ones, and the amount of parameters is notably reduced. However, they still present a large amount of non-formalized parameters. One gets the uncomfortable feeling that the variants originate in computational concerns: They are not necessarily improvements of the original Retinex model.

Horn's work¹¹ is probably the most remarkable modeling alternative to Retinex. Horn proposed to separate reflectance from illumination by taking the logarithm of the image intensity, $\log I = \log R + \log L$. Applying the Laplacian yields $\Delta \log I = \Delta \log R + \Delta \log L$. In order to eliminate the illumination component, Horn proposed to apply a threshold operator δ to remove it. This yields $\delta(\Delta \log I) \simeq \Delta \log R$. This relation can be viewed as a Poisson equation to find $\log R$ from I , namely

$$\Delta(\log R) = \delta(\Delta \log I). \quad (4)$$

This Poisson equation was solved by an iterative scheme, certainly not an optimal method for this equation. Blake¹² introduced an improvement to Horn's method. He pointed out that Horn's model is not exactly equivalent to the Land Retinex computation, and therefore proposed to threshold the gradient of the intensity image instead of its Laplacian.

There is also a variational Retinex theory proposed by Kimmel et al.¹⁵ They proposed to treat the illumination estimation as a quadratic programming optimization problem. The computational complexity is high and in a later paper³³ several methods have been proposed to tune down this complexity.

3. FAST IMPLEMENTATION OF RETINEX THEORY

The Retinex implementations have a high complexity and depend on a large number of parameters. In this section we present a fast implementation of Land's original Retinex algorithm. It has only one parameter, namely the threshold for δ in (3). This is the only parameter of the original Retinex. This implementation is based on the recent paper,²¹ where the original Retinex theory^{8,27} explained and defined in Section 2 was formalized as a Poisson equation. We shall use the following theorem, proved in the mentioned paper.²¹

Theorem 1. Define

$$F(x) = f\left(\frac{I(x)}{I(x_{-0})}\right) + f\left(\frac{I(x)}{I(x_{+0})}\right) + f\left(\frac{I(x)}{I(x_{0-})}\right) + f\left(\frac{I(x)}{I(x_{0+})}\right). \quad (5)$$

where x_{-0} , x_{+0} , x_{0-} and x_{0+} represent the four discrete x -neighbors, and $f(x) = \delta(\log x)$ with δ defined in (3). The lightness value in a chromatic channel L defined in Definition 1 is the unique solution of the discrete Poisson equation with Neumann boundary condition,

$$\begin{cases} -\Delta_d L(x) = F(x) & x \in \Omega \\ \frac{\partial L}{\partial n} = 0 & x \in \partial\Omega \end{cases}, \quad (6)$$

where F is a function that only depends on the intensity value at each pixel and its neighbors.

Remark 1. If we take in (3) $\delta(s) = s$ then the function F in (5) becomes $\Delta_d \log(I(x))$ and the equation (6) becomes

$$\Delta_d L(x) = \Delta_d \log(I(x)), \quad (7)$$

which also is Horn's equation (4) without threshold. This equation is, in some sense, equivalent to (6), since the threshold used in the definition of δ eliminates the small variations, which can be considered due to the illumination as we explained in Section 2.1.

Remark 2. Blake proposed to threshold the norm of the gradient intensity and his equation can be written

$$\text{div}(\nabla(\log R)) = \text{div}(\delta(\|\nabla \log I\|)\nabla \log I), \quad (8)$$

with δ defined as (3), which also yields a Poisson equation to find $\log R$ from I . In summary, there are three color constancy models formalized as a Poisson equation. They are very similar and only differ in their second member.

3.1 Fast Fourier Transform Implementation

The Poisson equation can be solved using the Fourier transform. The discrete Poisson problem (6) is easily solved with the discrete Fourier transform. The discrete Fourier transform of a two-dimensional function $f(n, m)$ on a $N \times M$ grid is defined by

$$\hat{f}(k, l) = \frac{1}{NM} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} f(n, m) e^{-i\frac{2\pi kn}{N}} e^{-i\frac{2\pi lm}{M}} \quad \begin{array}{l} k = 0, 1, \dots, N-1, \\ l = 0, 1, \dots, M-1 \end{array} \quad (9)$$

and the discrete inverse Fourier transform by

$$f(n, m) = \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \hat{f}(k, l) e^{i\frac{2\pi kn}{N}} e^{i\frac{2\pi lm}{M}} \quad \begin{array}{l} n = 0, 1, \dots, N-1, \\ m = 0, 1, \dots, M-1 \end{array} \quad (10)$$

The discrete Fourier transform has the following property

$$f(n - n_0, m - m_0) = \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \hat{g}(k, l) e^{i\frac{2\pi kn}{N}} e^{i\frac{2\pi lm}{M}} \quad \begin{array}{l} n = 0, 1, \dots, N-1, \\ m = 0, 1, \dots, M-1 \end{array} \quad (11)$$

where

$$\hat{g}(k, l) = \hat{f}(k, l) e^{-i\frac{2\pi kn_0}{N}} e^{-i\frac{2\pi lm_0}{M}}. \quad (12)$$

The discrete Poisson equation,

$$\Delta_d L(n, m) = F(n, m), \quad (13)$$

is solved by substituting the discrete Fourier transform in (13) and using the property (11), which yields

$$\hat{L}(k, l) \left(4 - 2 \cos \frac{2\pi k}{N} - 2 \cos \frac{2\pi l}{M} \right) = \hat{F}(k, l). \quad (14)$$

From (9) follows that $\hat{f}(0, 0)$ is the mean value of f on the grid. Our Poisson equation with Neumann boundary condition has a unique solution up to an additive constant. Thus, computing the solution whose mean is zero and using (14) yields

$$\hat{L}(k, l) = \begin{cases} \frac{\hat{F}(k, l)}{4 - 2 \cos \frac{2\pi k}{N} - 2 \cos \frac{2\pi l}{M}} & \text{if } (k, l) \neq (0, 0) \\ 0 & \text{if } (k, l) = (0, 0) \end{cases}. \quad (15)$$

Using (10), L can be computed at each point of the grid. Normalizing the range of L to the interval $[0, 255]$ yields the final solution. The Neumann boundary conditions are implicitly translated by making a symmetry along the image sides to obtain a $2N \times 2M$ image. All above computations are therefore performed the $2N \times 2M$ grid. This FFT implementation requires two FFTs on the image plus 13 operations per pixel, which amounts to 50 software operations/pixel. The lowest complexity path based method (Provenzi et al.²⁰) has minimal 3000 operations/pixel (but it is no more Retinex). Horn and Blake proposed to solve their Poisson equations by an iterative method which is $O(N^2 \times M^2)$. In contrast, on a 1 Million pixels image, the actual implementation time on standard PC is 0,13 seconds.

4. RETINEX AS IMAGE ENHANCEMENT

This section introduces a modification of the original Retinex by adding an upper threshold to the function $f(x) = \delta(\log(x))$. The aim is to increase the image contrast. We shall see that with this upper threshold added, Retinex becomes a good color-constancy tool.

4.1 The gamma-corrected model

The experiments on Retinex theory use the above-described discrete Poisson equation and its FFT solution described in Section 3. In principle, Retinex theory should be applied to raw images. Since it is difficult to display these images, the Retinex algorithm will be adapted to gamma-corrected images. Most of these images have already been used in articles on color correction. The gamma-correction consists of applying a concave function to the raw image, in practice a logarithm or a s^γ power with $0 < \gamma < 1$. Assuming that the gamma-correction is logarithmic is not restrictive: s^γ and \log have a very similar shape over the usual image range. As a rewarding consequence of this assumption, instead of working with differences between logarithms of intensities, we can deal directly with intensity differences. Thus we can write directly $f(x) = \delta(x)$, instead of $f(x) = \delta(\log(x))$, with δ defined by (3). Then the function F defined in (5) simply becomes

$$F(x) = f(I(x) - I(x_{-0})) + f(I(x) - I(x_{+0})) + f(I(x) - I(x_{0-})) + f(I(x) - I(x_{0+})). \quad (16)$$

4.2 Thresholds in Retinex

With the function F defined in (16), the second term of the Poisson equation (6) behaves as an edge detector and produces a positive impulse located on the brighter side of the edge and a negative impulse located on the darker side. The good point of expressing Land's model on the gamma-corrected image is that we can gain an intuitive meaning for the Retinex threshold t used in the definition of δ in (3). This threshold allows to eliminate the small impulses. Now, in gamma-corrected images edges are usually perceived when their gradient exceeds a value around 10 (for classic 0-255 ranges). Thus, to avoid squeezing contrast in the image the lower threshold t must be conspicuously smaller than 10.

We shall now introduce an upper threshold which is not present in the Retinex theory. This is the only alteration to the original theory that will be tried. It is strongly motivated by computer vision visualization concerns that exceed the original Retinex scopes. Indeed, Retinex did not deal with digital images and admitted arbitrary image ranges. Now, computerized images do have a range constraint, typically $[0, 255]$. Thus, the algorithm must take the best advantage of this range by allowing a reduction of strong edges, provided such edges are still perceived as edges. This is obtained by introducing an upper threshold T which reduces very large impulses and gives in practice more freedom for contrast adjustment. The definition of the function f is modified as

$$f(x) = \begin{cases} 0 & \text{if } |x| < t \\ x & \text{if } t < |x| < T \\ \text{sign}(x)T & \text{if } |x| > T \end{cases} . \quad (17)$$

We will call this new model *Contrast Retinex*, since it manipulates the image high contrasts, and not just the low ones. In all experiments we have linearly normalized each channel so that its standard deviation is constant and equal to 40. Otherwise, thresholds would be highly dependent on the channel contrast and wouldn't be universal. Once this is done, the sound thresholds are $t = 3$, which is reasonable if we want to eliminate the minor variations of the intensity, and $T = 20$ which is a sufficiently large cut-off for maintaining edges.

4.3 Experimental Results

To understand the effect of the thresholds under discussion, Figures 1, 2 show the original image and the result using various values of t and T . Figure 1 shows the Contrast Retinex result with $t = 0, 3, 5$. Notice how the clarity of the background is affected by the lower threshold value. Figure 2 shows the result with $T = 20$ and without upper threshold T (original Retinex). The result with the upper threshold is definitely more contrasted. Contrast Retinex enhances images, as can be seen in Figs. 2 and 3.

As we have mentioned in the introduction Land's Retinex is the first computational model for color constancy. With Contrast Retinex, the lightness seems to be well recovered in spite of strong scene illumination changes.

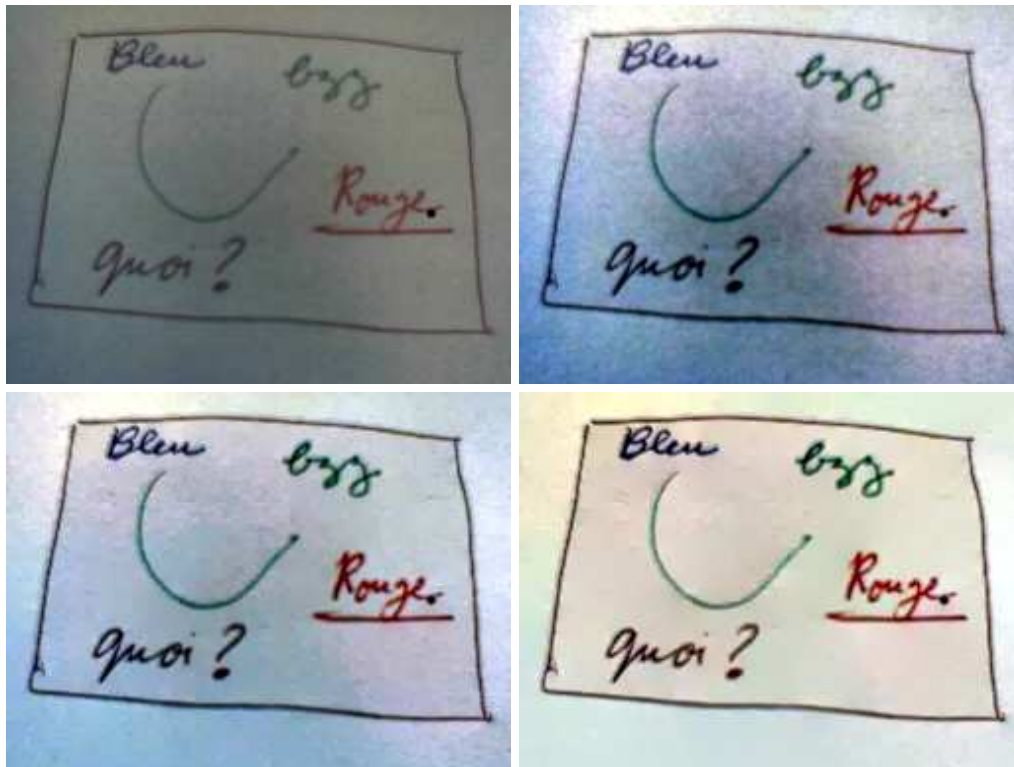


Figure 1. Top Left: original image. Top Right: result of Contrast Retinex with $t = 0$ and $T = 20$. Bottom: results with $t = 3$ and $t = 5$ respectively with $T = 20$. The first experiment illustrates the role of the higher threshold to enhance the general image contrast. The other two experiments show how texture is eliminated and color enhanced by the original Retinex lower threshold



Figure 2. Left: original image. Middle: Contrast Retinex result with $t = 3$ and $T = 20$. Right: result without T , original Retinex, $t = 3$. Again, this experiment shows the usefulness of the higher contrast threshold.



Figure 3. Left: original image. Right: result of Contrast Retinex with $t = 1$ and $T = 40$.

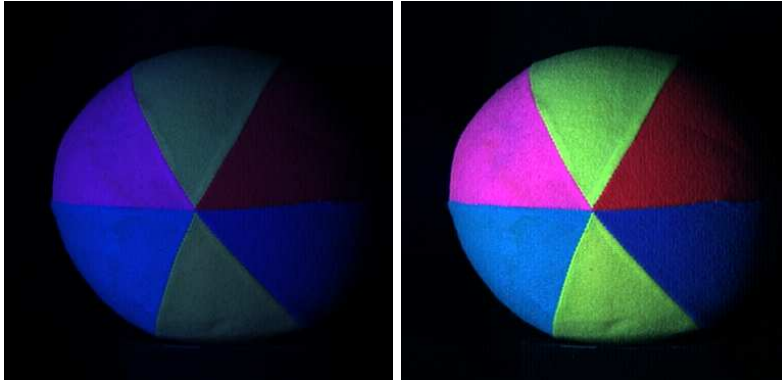


Figure 4. Left: original image. Right: result of Contrast Retinex with $t = 1$ and $T = 40$.

Figure 4 shows a ball under a blue illumination and the result after applying Contrast Retinex. Figure 5 shows a photograph of a film, a reddish face image, and the result after Retinex.

We pointed out that in its Poisson formulation Land's Retinex turns out to be very similar to the Horn and Blake models. These models can be implemented using the fast algorithm of Section 3.1 as well. Horn's model thresholds the laplacian of the intensity in order to eliminate the illumination effects. Figure 6 shows the result on the image in Figure 1 with Horn's equation and a lower threshold equal to $t = 0.5$ and $t = 2$ respectively. Contrary to the original Retinex model (with lower threshold), increasing the lower threshold of the laplacian



Figure 5. Left: original image. Right: result of Contrast Retinex with $t = 1$ and $T = 20$.

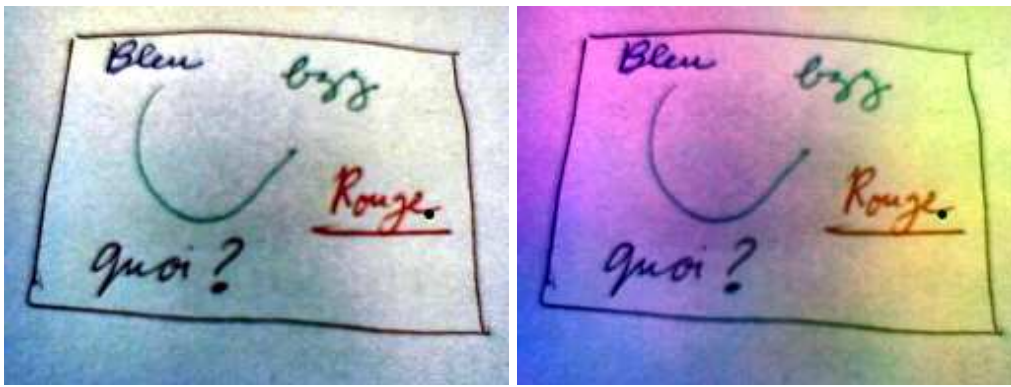


Figure 6. Result of Horn's model with lower thresholds 0.5 and 2.

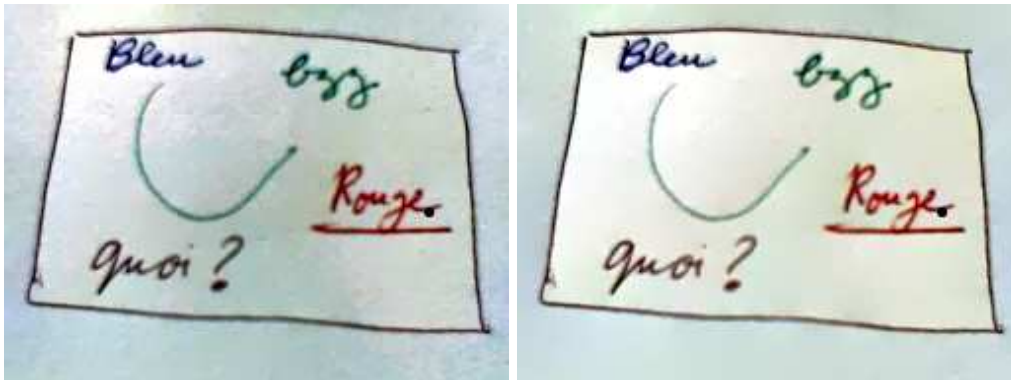


Figure 7. Result of Blake's model with lower thresholds 18 and 50.

leads to the production of serious color artifacts. In Horn's model the choice of the threshold is therefore crucial for the final result.

Blake's model applies a lower threshold to the norm of the intensity gradient. In that sense, it is more similar to Retinex, but it is isotropic, while the original Retinex is anisotropic, the thresholds being applied to the intensity variation in each direction. Figure 7 shows the result with two different thresholds $T = 18$ and $T = 50$. Increasing the threshold produces a filtering effect analogous to the Retinex result in Figure 1.

Since the upper threshold improves the results as showed in Figure 2, an upper threshold has been tried also in the Horn and the Blake models. Horn's model does not work with upper threshold as can be seen in Figure 8 left. Blake's model instead proves similar to Contrast Retinex. Figure 8 right shows the result of Blake with a lower threshold 5 and an upper threshold 20.

5. CONCLUSION

A fast implementation of several classic color constancy algorithms using the Fast Fourier Transform has been described. This has permitted to compare the Land-McCann, Horn, and Blake's models in a unified formalism and with the very same algorithm, and to compare experimentally their results. The final formalism only involves two intuitive parameters, namely the lower and higher thresholds for edge preservation. Although Retinex aimed essentially at color balance, the experiments confirm that the upper threshold proposed here provides an excellent local contrast adjustment, in the spirit of.³⁴ One of the outcomes of the comparison is that the Retinex model is superior to Horn's and Blake's model.

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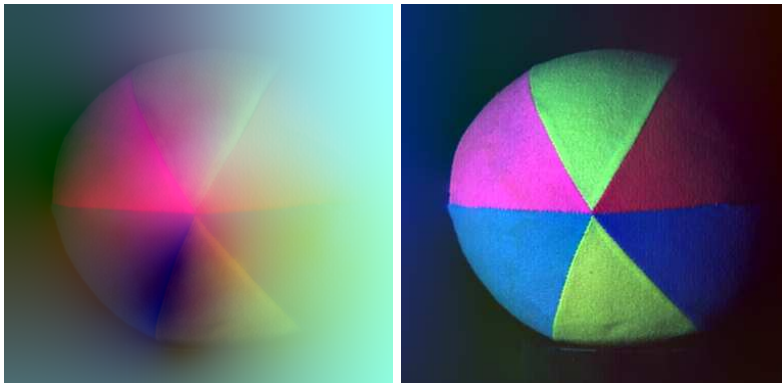


Figure 8. Left: Horn result with upper threshold. Right: Blake's result with lower threshold $t = 5$ and upper threshold $T = 20$.

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