ASIFT:
A New Framework for Fully Affine Invariant Image Comparison

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Proposed method: ASIFT.
State of the art: SIFT.
State of the art: MSER.
State of the art: Hessian Affine.
State of the art: Harris Affine.
Proposed method: ASIFT.
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The new state of the art:

It is by now possible to recognize a solid object in a digital image, no matter what the angle and the distance, up to limits that only depend on resolution.
In this pair: A very large transition tilt (extreme angle). The transition tilt will be defined later.
90 correct matches, 4 outliers. The matches were obtained by the \textit{Affine SIFT} method (ASIFT), a variant of the SIFT method.
The projective camera model $u = S_1 G_1 A u_0$.  
- $A$ is a planar projective transform (homography).  
- $G_1$ is an anti-aliasing gaussian filter.  
- $S_1$ is the CCD sampling. Shannon condition satisfied: $u = S_1 G_1 A u_0 \rightarrow u = G_1 A u_0$.  

\[
\begin{align*}
u &= S_1 G_1 A u_0 \\
y &\text{digital image} & S_1 & \text{sampling (grid)} & G_1 & \text{Gaussian kernel (blur)} & A & \text{planar projective map} & u_0 & \text{original infinite resolution surface}
\end{align*}
\]
Affine Simplification

If the object’s shape is locally smooth, local deformations in a single view can be approximated by several different local affine transforms.

Affine transforms map rectangles to parallelograms.
Geometric Interpretation of the Six Affine Parameters

\[ u = S_1 G_1 A u_0. \]

\( A \) is an affine map:

\[
\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}
\]

\[
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = H_\lambda R_1(\psi) T_\theta R_2(\phi) = \lambda \begin{bmatrix} \cos \psi & -\sin \psi \\
\sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} t & 0 \\
0 & 1 \end{bmatrix} \begin{bmatrix} \cos \phi & -\sin \phi \\
\sin \phi & \cos \phi \end{bmatrix}
\]

- \( \phi \): longitude angle between optical axis and a fixed vertical plane.
- \( \theta = \arccos(1/t) \): latitude angle between optical axis and the normal to the image plane.
  \( \text{Tilt } t > 1 \iff \theta \in [0^\circ, 90^\circ] \).
- \( \psi \): rotation angle of camera around optical axis.
- \( \lambda \): zoom parameter.
- \( T = (e, f)^T \): translation, not presented here.
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- \( \theta = \arccos(1/t) \): latitude angle between optical axis and the normal to the image plane.
  
  Tilt \( t > 1 \leftrightarrow \theta \in [0^\circ, 90^\circ] \).
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- \( \psi \): rotation angle of camera around optical axis.
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\[ \text{\( \lambda \)}: \ text{\textit{zoom}} \text{ parameter.} \]
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\begin{align*}
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  A &\text{ is an affine map: } \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \\
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\end{align*}
\]

- \( \lambda \): zoom parameter.
Transition Tilts

Both compared images are usually slanted views. The transition tilt quantifies the tilt between two such images.

**Definition** Consider two views of a planar image, \( u_1(x, y) = u(A(x, y)) \) and \( u_2(x, y) = u(B(x, y)) \) where \( A \) and \( B \) are two linear maps such that \( BA^{-1} \) is not a similarity. We call transition tilt \( \tau(u_1, u_2) \) and transition rotation \( \phi(u_1, u_2) \) the unique parameters such that

\[
BA^{-1} = H_\lambda R_1(\psi)T_\tau R_2(\phi). \tag{1}
\]
Properties of Transition Tilts

- The transition tilt is symmetric, i.e., \( \tau(u_1, u_2) = \tau(u_2, u_1) \);

- The transition tilt only depends on the absolute tilts and on the longitude angle difference: \( \tau(u_1, u_2) = \tau(t, t', \phi - \phi') \);

- One has \( t' / t \leq \tau \leq t' t \), assuming \( t' = \max(t', t) \);

- The transition tilt is equal to the absolute tilt: \( \tau = t' \), if the other image is in frontal view (\( t = 1 \)).
High Transition Tilts

\[ \tau = 36 \Rightarrow \theta = 88.41^\circ \]
High Transition Tilts

\[ \tau < 2 \text{ (SIFT)} \quad \tau < 10 \text{ (MSER)} \quad \tau < 40 \text{ (ASIFT)} \]

\[ \theta = 80^\circ \]
Affine Invariance: Simulation v.s. Normalization

- **Simulation.**
  - all 6 parameters impossible, e.g. $10^6$.

- **Normalization.**

  \[ u = G_1 A u_0, \quad v = G_1 u_0 \Rightarrow u = A v \]

**Non-commutation:** in general, $G_1 A u_0 \neq AG_1 u_0$

- Translation $T$ and rotation $R$ can be normalized.
  **Strong commutation** with blur $\Rightarrow$ normalization possible.

- Zoom $H_\lambda$ and tilt $T$ cannot be normalized *stricto sensu.*
  **Weak commutation** with blur $\Rightarrow$ simulation necessary.

\[ H_\lambda G_1 = G_{1/\lambda} H_\lambda \Rightarrow H_\lambda v \neq u \]
Affine Invariance: Simulation v.s. Normalization
State-of-the-art

- SIFT (Scale-Invariant Feature Transform) [Lowe 99, 04]:
  - Rotation and translation are normalized.
  - Zoom is simulated in the scale space.
  - No treatment on latitude and longitude: modest robustness $\tau_{\text{max}} < 2$.

- MSER (Maximally Stable Extremal Region) [Matas et al. 02] and LLD (Level Line Descriptor) [Musé et al. 06]
  - Attempt to normalize all the parameters.
  - Weakness: limited affine invariance $\tau_{\text{max}} < 10$, not scale invariant, small number of features.

- Other methods: Harris-Affine, Hessian-Affine [Mikolajczyk and Schmid 04]
State-of-the-art

- Other methods: [Baumberg, 00; Tuytelaars and Van Gool, 00, 04; Mikolajczyk and Schmid, 02, 04, 05; Schaffalitzky and Zisserman, 02; Brown and Lowe, 02, S. Belongie, J. Malik, and J. Puzicha, 02, Kadir, Zisserman, Brady, 04, Ke and Sukthankar, 04]

  - SIFT-based descriptors perform best.
  - MSER outperforms other affine invariant detectors such as Hessian Affine and Harris Affine.
SIFT: Scale Invariant Features Transform

- the initial digital image is $S_1G_1Au_0$, $A$ is any similarity, $u_0$ is the underlying infinite resolution planar image;
- at all scales $\sigma > 0$, the SIFT method computes $u(\sigma, \cdot) = G_\sigma G_1 Au_0$ and “key points” $(\sigma, x)$, namely scale and space extrema of $\Delta u(\sigma, \cdot)$;
- the blurred $u(\sigma, \cdot)$ image is sampled around each key point at a pace proportional to $\sqrt{1 + \sigma^2}$;
- directions of the sampling axes are fixed by a dominant direction of $\nabla u(\sigma, \cdot)$ in a $\sigma$-neighborhood of the key point;
- this yields rotation, translation and scale invariant samples: the 4 parameters of $A$ have been eliminated!;
- the final SIFT descriptor keeps only orientations of the gradient to gain invariance w.r. light conditions.
SIFT Feature Points
SIFT: Scale Invariant Features Transform

Each key-point is associated a square image patch whose size is proportional to the scale and whose side direction is given by the assigned direction. Example of a $2 \times 2$ descriptor array of orientation histograms (right) computed from an $8 \times 8$ set of samples (left). The orientation histograms are quantized into 8 directions and the length of each arrow corresponds to the magnitude of the histogram entry.
Affine-SIFT (ASIFT) Overview

- Simulate latitude, longitude to achieve full affine invariance.
- Simulated images are compared by a rotation-, translation- and zoom-invariant algorithm, e.g., SIFT. (SIFT normalizes translation and rotation and simulates zoom.)
Inverting Tilts

**Definition** Given $t > 1$, the tilt factor, define

- the **geometric** tilt: $T^x_t u_0(x, y) := u_0(tx, y)$.  
  In the $y$ direction, $T^y_t u_0(x, y) := u_0(x, ty)$.

- the **simulated** tilt (taking into account camera blur): $\mathbb{T}^x_t v := T^x_t G^x_t \sqrt{t^2 - 1} \ast_x v$.  
  In the $y$ direction, $\mathbb{T}^y_t v := T^y_t G^y_t \sqrt{t^2 - 1} \ast_y v$.

- **Main Formula**  
  For $t \geq 1$, $\mathbb{T}^y_t G_1 T^x_t = G_1 H_t$.  
  Geometric tilts in $x$ are reversed by simulated tilts in $y$ up to a zoom-out scale change.
ASIFT Algorithm

1. Apply a dense set of rotations to both images $u$ and $v$.

2. Apply in continuation a dense set of simulated tilts $T^x_t$ to all rotated images.

3. Perform a SIFT comparison of all pairs of resulting images.

Notice that by the relation

$$T^x_t R \left( \frac{\pi}{2} \right) = R \left( \frac{\pi}{2} \right) T^y_t,$$

ASIFT simulates tilts in the $y$ direction, up to a rotation.
Consistency of ASIFT: reduction from ASIFT to SIFT

**Theorem 1** Let \( u = G_1 A T_1 u_0 \) and \( v = G_1 B T_2 u_0 \) be two images obtained from an infinite resolution image \( u_0 \) by cameras at infinity with arbitrary position and focal lengths. Then ASIFT, applied with a dense set of tilts and longitudes, simulates two views of \( u \) and \( v \) that are obtained from each other by a translation, a rotation, and a camera zoom. As a consequence, these images match by the SIFT algorithm.
Proof that ASIFT works

\[ BA^{-1} = H_\lambda R_1 T_t^x R_2. \]

Compare: \( u = G_1 u_0, \quad v = G_1 R_1 T_t^x R_2 H_\lambda u_0. \)

Applying \( R_1^{-1} \) to \( v \) yields \( v \to v' = G_1 T_t^x R_2 H_\lambda u_0. \)

Then revert \( T_t^x \) by applying the simulated tilt in the \( y \) direction to \( v' \):

\( T^y := T_t^y G_1^y \sqrt{t^2-1} y. \) Indeed (main formula):

\[ T_t^y G_1 T_t^x = G_1 H_t. \]

Thus by application of \( T^y \) to \( v' \) we get

\[ v' \to G_1 H_t R_2 H_\lambda u_0 = G_1 H_{t\lambda} R_2 u_0, \]

which is SIFT equivalent to \( u \).
Parameter Sampling Range

- Longitude angle $\phi \in [0, \pi)$.
  
  $\mathbf{R}_1(\psi)\mathbf{T}_t\mathbf{R}_2(\phi + \pi) = \mathbf{R}_1(\psi + \pi)\mathbf{T}_t\mathbf{R}_2(\phi)$.

- Tilt $t = 1/\cos \theta \in [1, t_{\text{max}}]$.
  - Physical limitation: planar and Lambertian.
  - $t_{\text{max}} = 4\sqrt{2}$ obtained experimentally.
  - The resulting $\tau_{\text{max}} = 32$. 
Parameter Sampling Range

\[ t = 3 \ (\theta = 70.5^\circ), \ 151 \text{ correct ASIFT matches.} \]
Parameter Sampling Range

t = 5.2 (θ = 78.9°), 12 correct ASIFT matches.
Parameter Sampling Range

t = 8 (θ = 82.8°), 0 correct match.
Parameter Sampling Range

\[ t = 3.8 \ (\theta = 74.7^\circ), \ 116 \ \text{correct ASIFT matches}. \]
Parameter Sampling Range

\[ t = 5.6 \ (\theta = 79.7^\circ) \], 26 correct ASIFT matches.
Parameter Sampling Range

\[ t = 8 \ (\theta = 82.8^\circ), \ 0 \ \text{ASIFT match}. \]
Parameter Sampling Step: $\Delta t$

- $t = 1/\cos \theta$, $\theta$ is the latitude angle.
- $\theta$ sampled with higher precision when $\theta \to 90^\circ$.
- Geometric sampling of $t$: $\Delta t = t_{k+1}/t_k$.
- $\Delta t = \sqrt{2}$ is obtained experimentally: compare $\mathbf{u} = T_{t_1} \mathbf{u}_0$ and $\mathbf{v} = T_{t_2} \mathbf{u}_0$ with SIFT.

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>$\sqrt{2}$</th>
<th>2</th>
<th>$2\sqrt{2}$</th>
<th>4</th>
<th>$4\sqrt{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>$0^\circ$</td>
<td>$45^\circ$</td>
<td>$60^\circ$</td>
<td>$69.3^\circ$</td>
<td>$75.5^\circ$</td>
<td>$79.8^\circ$</td>
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</tbody>
</table>
Parameter Sampling Step: $\Delta \phi$

- $\phi$: longitude angle.
- $\phi$ sampled with higher precision when $\theta \to 90^\circ$: $t \uparrow \Rightarrow \Delta \phi \downarrow$.
- Arithmetical sampling of $\phi$: $\Delta \phi = \phi_{k+1} - \phi_k$.
- $\Delta \phi = 2 \times \frac{36^\circ}{t} = \frac{72^\circ}{t}$ is obtained experimentally: compare $\mathbf{u} = T_t R_1(\phi) \mathbf{u}_0$ and $\mathbf{v} = T_t \mathbf{u}_0$ with SIFT.
Parameter Sampling

Perspective view

View from the zenith
Acceleration: Multi-resolution ASIFT

1. ASIFT on low-resolution images \((r \times r)\) sub-sampled.

2. ASIFT on high-resolution images obtained with the M best affine transforms (only in case of success in 1.).
ASIFT Complexity

- Complexity proportional to (area of query) \( \times \) (searched area).

- Image area proportional to number of simulated tilts.
  - \( t = 2^{k/2}, \ k = 0, \ldots, K \).
  - \( \phi \in [0^\circ, 180^\circ), \ \triangle \phi = \frac{72^\circ}{t} \):
    \(|\{\phi(t)\}| \sim t\).
  - At tilt \( t \), image area \( \sim 1/t \).

- Example: \( t_{\text{max}} = 4\sqrt{2} \) (i.e. \( K = 5 \), \( r \times r = 3 \times 3 \) subsampling.
  - Image area on one side:
    \[
    \frac{1 + K \frac{180^\circ}{72^\circ}}{r^2} = 1.5 \times \text{original image}
    \]
  - One sided ASIFT (tilts simulated on query only): total complexity = \( 1.5 \times \text{SIFT} \), \( \tau_{\text{max}} = 4\sqrt{2} \approx 5.6 \).
  - Two sided ASIFT (tilts simulated on query and searched images): total complexity = \( (1.5)^2 \times \text{SIFT} = 2.25 \times \text{SIFT} \), \( \tau_{\text{max}} = 32 \).
Experiments: Image Matching

Zoom change. Number of correct matches: ASIFT (left)—222; SIFT (middle)—87; MSER (right)—4.
Experiments: Image Matching

Frontal v.s. $-45^\circ$ angle, zoom $\times1$: absolute tilt $t = 2$ (middle), $t < 2$ (left part), $t > 2$ (right part). Number of correct matches: ASIFT (left)—624; SIFT (middle)—236; MSER (right)—11.
Experiments: Image Matching

Frontal v.s. 75° angle, zoom ×1: absolute tilt $t = 4$ (middle), $t < 4$ (left part), $t > 4$ (right part). Number of correct matches: ASIFT (left)—202; SIFT (middle)—15; MSER (right)—5.
Experiments: Image Matching

Frontal v.s. $-80^\circ$ angle, zoom $\times 10$: absolute tilt $t = 5.8$. Number of correct matches: ASIFT (left)—75; SIFT (middle)—1; MSER (right)—2.
Experiments: Image Matching

Correspondences between the magazine images taken with absolute tilts $t_1 = t_2 = 2$ with longitude angles $\phi_1 = 0^\circ$ and $\phi_2 = 50^\circ$, transition tilt $\tau = 3$. Number of correct matches: ASIFT (left)—881; SIFT (middle)—2; MSER (right)—87.
Experiments: Image Matching

Correspondences between the magazine images taken with absolute tilts \( t_1 = t_2 = 4 \) with longitude angles \( \phi_1 = 0^\circ \) and \( \phi_2 = 90^\circ \), transition tilt \( \tau = 16 \). Number of correct matches: ASIFT (left)—88; SIFT (middle)—1; MSER (right)—9.
Experiments: Image Matching

Graffiti 1 vs 6.
Transition tilt: $\tau \approx 3.2$.
Number of correct matches:
ASIFT (top)—721;
SIFT (middle)—0;
MSER (bottom)—70.
Experiments: Image Matching

Images proposed by Matas et al.
Number of correct matches:
ASIFT (top)—254;
SIFT (middle)—10;
MSER (bottom)—22.
Experiments: Image Matching

Road signs.
Transition tilt: \( \tau \approx 2.6 \).

Number of correct matches:
ASIFT (top)—50;
SIFT (middle)—0;
MSER (bottom)—1.
Experiments: Image Matching

Parkings.
Transition tilt: $\tau \approx 15$.

Number of correct matches:
ASIFT (top)—78;
SIFT (middle)—0;
MSER (bottom)—0.
Experiments: Image Matching

Ecole Polytechnique.

Transition tilt $\tau = 2.4$. Number of correct matches: ASIFT (left)—103; SIFT (middle)—13; MSER (right)—4.
Experiments: Image Matching

Stump. Transition tilt $\tau = 2.6$. Number of correct matches: ASIFT (left)—168; SIFT (middle)—1; MSER (right)—6.
Experiments: Image Matching

Pentagon. Transition tilt $\tau \approx 2.5$.

Number of correct matches: ASIFT (left)—378, SIFT (middle)—6, MSER(right)—17.
Experiments: Image Matching

Statue of Liberty. Transition tilt $\tau \in [1.3, \infty)$. 

Number of correct matches: ASIFT (left)—22, SIFT (right)—1.
Experiments: Image Matching

Left: flag. ASIFT (shown)—141, SIFT—31, MSER—2.
Right: SpongeBob. ASIFT (shown)—370, SIFT—75, MSER—4.
Experiments: Object Tracking
Symmetry Detection in Perspective

Symmetry detection = image comparison with its flipped version.

ASIFT  SIFT  MSER
Summary: fully affine-invariant image comparison

- Camera interpretation of affine space: 6 parameters.
- High transition tilts.
- Simulation v.s. normalization.
- Simulate scale, longitude and latitude.
- Normalize translation and rotation.
- Mathematical proof: fully affine-invariant.
- Sample the camera hemisphere (longitude and latitude).
- Multi-resolution acceleration.
- Reasonably small complexity.
- State-of-the-art results.
Reference:


Patent:


Website and Online Demo: try ASIFT with your images!

- [http://www.cmap.polytechnique.fr/~yu/research/ASIFT/demo.html](http://www.cmap.polytechnique.fr/~yu/research/ASIFT/demo.html)

For more information,

Google ASIFT