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# Implementation of the Midway Image Equalization

Thierry Guillemot, Julie Delon

MAP5, Université Paris Descartes, France
(thierry.guillemot.work@gmail.com, julie.delon@parisdescartes.fr)

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**Abstract**

In this paper, we present the detailed algorithm of the Midway Image Equalization giving to a pair of images the same histogram while maintaining as much as possible their previous gray dynamics. The midway equalization is primarily designed for gray level images, but can be applied channel-wise to color images. This method is easy to implement, fast to compute, fully automatic and requires no parameter.

**Source Code**

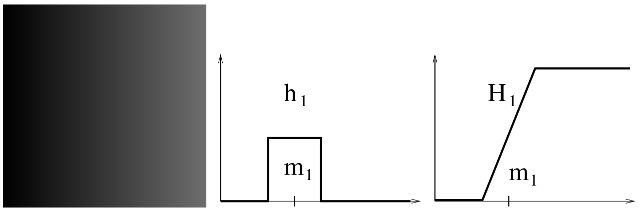
The source code and an online demo are accessible at the IPOL web page of this article[1]. The documentation is included in the archive. Basic compilation and usage instructions are included in the README.txt file. The demo permits to try the proposed method on several test images, and on any uploaded couple of images. A common drawback of the methods aiming at modifying the contrast of images is their tendency to create quantization artifacts. To reduce the quantization effect of the equalized images, we propose as an option to use the dithering method. In that case, it will be necessary to give the Gaussian noise standard deviation value as parameter. The outputs are the processed images.

**Keywords:** histogram; equalization; image comparison

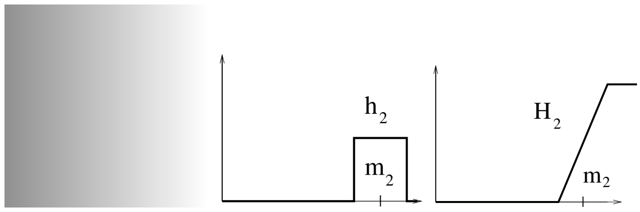
## 1 Introduction

The idea of the midway equalization [2] is to give to a pair of images the same intermediate histogram (histogram should be understood here in the sense of gray level or color distribution) while preserving as much as possible their previous gray level or color dynamics. An early version of this algorithm was introduced by Cox and al. [1] under the name of Dynamic histogram warping. Computing such midway equalizations is particularly useful for the comparison of images of the same scene, but can also be applied to collections of images for applications such as flicker reduction [3, 4], for instance.

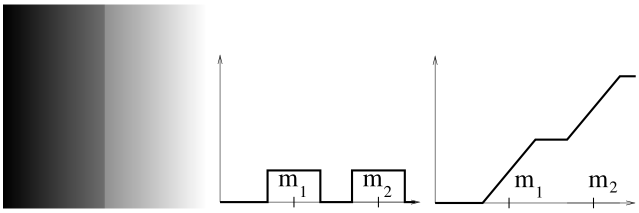
[1] https://doi.org/10.5201/ipol.2016.140



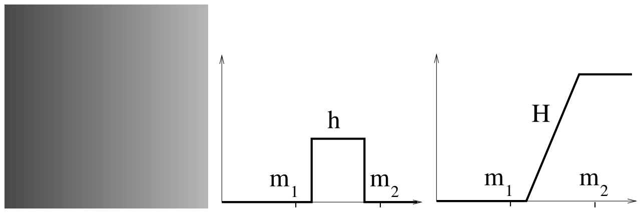

(a) Image $u_1$, its histogram $h_1$ and its cumulative histogram $H_1$.




(b) Image $u_2$, its histogram $h_2$ and its cumulative histogram $H_2$.




(c) Resulting image by averaging the histograms $h_1$ and $h_2$.




(d) Resulting image by applying a *midway* equalization

Figure 1: Comparisons between the arithmetic and the harmonic average of the cumulative histograms for two gradient images.

Recall that if $u$ is a digital image defined on the discrete grid $\Omega = \{0, \ldots, N-1\} \times \{0, \ldots, M-1\}$, the cumulative histogram $H_u$ of $u$ is defined as

$$H_u(\lambda) = \frac{1}{|\Omega|} |\{(i,j) \in \Omega;\ u(i,j) \leq \lambda\}|,$$

where $|\Omega| = N \times M$ is the number of pixels of $u$. If $u$ takes its values in the set $\{0, \ldots, L\}$, the discrete histogram $h_u$ of $u$ is the discrete derivative of $H_u$, i.e.

$$\forall l \in \{0, \ldots, L\},\ \ h_u(l) = \frac{1}{|\Omega|} |\{(i,j) \in \Omega;\ u(i,j) = l\}|.$$

Consider two discrete gray level images $u_1$, $u_2$ which take their values in the same set $\{0, \ldots, L\}$. Let $h_1$ and $h_2$ denote their respective normalized gray level histograms. In order to average the contrast between $u_1$ and $u_2$ (i.e. to specify their respective gray level histograms on a common one), a naive solution would be to apply a contrast change to each image such that the gray level distribution of both images would fit $(h_1 + h_2)/2$. As shown in Figure 1, such a solution is generally unsatisfactory: if $u_1$ (resp. $u_2$) has a unimodal gray level histogram centered at $m_1$ (resp. $m_2$), the average histogram $(h_1 + h_2)/2$ contains two modes, one at $m_1$ and the other at $m_2$. It would be much more natural to define a midway histogram between them as a unimodal histogram centered at $(m_1 + m_2)/2$.

As it was shown in [2], a satisfactory definition of the midway histogram between $h_1$ and $h_2$ is the harmonic mean between their cumulative histograms $H_1$ and $H_2$, i.e. $H_{midway} = (0.5(H_1^{-1} + H_2^{-1}))^{-1}$ (Figure 1). The resulting contrast changes $f_{12}$ and $f_{21}$ applied to the images $u_1$ and $u_2$ are

$$\begin{aligned} f_{12}(x) &= \frac{1}{2}(x + H_2^{-1} \circ H_1(x)), \\ f_{21}(x) &= \frac{1}{2}(x + H_1^{-1} \circ H_2(x)). \end{aligned}$$

The midway distribution possesses several properties:

- the images $\hat{u}_1 = 0.5(u_1 + H_2^{-1} \circ H_1(u_1))$ and $\hat{u}_2 = 0.5(u_2 + H_1^{-1} \circ H_2(u2))$ have the same cumulative histogram $H_{midway}$.

- if there exists an image $u_0$ and two contrast changes $f$ and $g$ such that $u_1 = f(u_0)$ and $u_2 = g(u_0)$, then $H_{midway}$ is the cumulative histogram of $(u_1 + u_2)/2$.

- gray levels having the same rank in both images are averaged together.

# 2 Implementation

## 2.1 Algorithm

**Algorithm 1:** General midway equalization

**input** : two discrete images $u_p(i,j) \in \{0,\dots,L\}$, $p \in \{1,2\}$ and $(i,j) \in \{1,\dots,N_p\} \times \{1,\dots,M_p\}$

**output**: two discrete images $\hat{u}_p(i,j)$

1 **for** *each* $p \in \{1,2\}$ **do**

2 compute the cumulative normalized histogram $H_p$ of the image $u_p$

$$\begin{aligned} H_p : \{0,\dots,L\} &\rightarrow [0,1] \\ l &\rightarrow H_p(l) = \frac{1}{N_p M_p} \sum_{k=0}^{l} \sum_{i,j} \mathbf{1}_{\{u_p(i,j)=k\}} \end{aligned}$$

3 **end**

4 compute the midway histogram $\tilde{H}^{-1}$

$$\begin{aligned} \tilde{H}^{-1} : [0,1] &\rightarrow \{0,\dots,L\} \\ x &\rightarrow \tilde{H}^{-1}(x) = \frac{1}{2} \sum_{p=1}^{2} H_p^{-1} \end{aligned}$$

**for** *each* $p \in \{1,2\}$ **do**

5 **for** *each pixel position* $(i,j) \in \{1,\dots,N_p\} \times \{1,\dots,M_p\}$ **do**

6 compute the corrected pixel $\hat{u}_p(i,j) \in \{0,\dots,L\}$

$$\hat{u}_p(i,j) = \tilde{H}^{-1} \circ H_p(u_p(i,j))$$

7 **end**

8 **end**

A first description of the midway algorithm for a pair of images is provided in Algorithm 1. In that way, the algorithm is quickly changeable to be applied to a group of several images (by changing the 2 by the number of images in step 1, 4, 5).

In practice, this algorithm can be easily simplified. For example, in the case of two images of the same size $N$, the computation can be done by sorting the gray levels of the two input images $u_1$ and

$u_2$ into two vectors of length $N$. The resulting equalization can be obtained by averaging these two ordered vectors and assigning to each pixel of each original image the gray level of the element in the averaged vector which has the same rank.

In Matlab, it can be computed by the following lines:

```
u_midway_1 = zeros(size(u_1));
u_midway_2 = zeros(size(u_2));

[u_sort_1,index_u_1] = sort(u_1(:));
[u_sort_2,index_u_2] = sort(u_2(:));
u_midway_1(index_u_1) = (u_sort_1 + u_sort_2)/2;
u_midway_2(index_u_2) = (u_sort_1 + u_sort_2)/2;

u_midway_1 = reshape(u_midway_1,size(u_1));
u_midway_2 = reshape(u_midway_2,size(u_2));
```

However, this simplified implementation suffers from some drawbacks:

- the algorithm can not be generalized for different image sizes
- two pixels with the same gray level on an original image can be transformed into two different gray levels.
- this method uses a sorting algorithm which increases significantly the computation time.

For these reasons, our implementation (shown in Algorithm 2) computes explicitly the equalized images $\hat{u}_1$, $\hat{u}_2$ from the cumulative histograms $H_1$, $H_2$ by using a lookup table to accelerate the computation of the contrast functions $f_{12}$ and $f_{21}$.

## 2.2 Color Images

Initially, the midway equalization was designed for gray level images. A fully satisfying extension to color images should involve optimal transportation (see [5, 6]). For the sake of simplicity, we propose in this demo to apply the midway equalization independently on each color channel. In figures 6-8, we present the resulting color midway equalization over several pairs of color images, applied separately on the R, G and B channels. Observe that this separate application of contrast changes on the three channels can lead to severe color artifacts when it is applied to images of different scenes (as shown in Figure 9).

## 2.3 Dithering

When images possess really different histograms, the equalized images can suffer from some quantization artifacts. Indeed, an information contained into few and close histogram bins can be transported into the same number of bins but not necessarily close to each other (as shown in Figure 5).

To visually improve the quality of the equalized images, we propose to remove the large-scale pattern introduced by using a dithering method. Thus, we add a Gaussian noise of standard deviation $\sigma$ to the original images before applying the midway equalization. For small values of $\sigma$ (e.g. $\sigma \approx 2$) we obtained good visual performances without introducing disturbing noise.

**Algorithm 2:** Implemented midway equalization

**input** : two discrete images $u_p(i,j) \in \{0,\dots,L\}$, $p \in \{1,2\}$ and $(i,j) \in \{1,\dots,N_p\} \times \{1,\dots,M_p\}$

**output**: two discrete images $\hat{u}_p(i,j)$

*// Computation of the cumulative histograms $H_1$ and $H_2$ of the images $u_1$ and $u_2$*

**for** *each* $p \in \{1,2\}$ **do**

  **for** *each pixel position* $(i,j) \in \{1,\dots,N_p\} \times \{1,\dots,M_p\}$ **do**

    $H_p(u_p(i,j)) = H_p(u_p(i,j)) + \frac{1}{N_p M_p}$

  **end**

  **for** *each bin position* $l \in \{1, L\}$ **do**

    $H_p(l) = H_p(l) + H_p(l-1)$

  **end**

**end**

*// Computation of the contrast function $f_{12}$ using a lookup table*

initialize bin position $l = 0$

**for** *each bin position* $k \in \{0,\dots,L\}$ **do**

  **while** $H_1(k) > H_2(l)$ **do**

    $l = l + 1$; *// Find the position $l$ where the values $H_1(k)$ and $H_2(l)$ are equal*

  **end**

  $f_{12}(k) = \frac{1}{2}(k + l)$

**end**

*// Computation of the contrast function $f_{21}$ using a lookup table*

initialize bin position $l = 0$

**for** *each bin position* $k \in \{0,\dots,L\}$ **do**

  **while** $H_2(k) > H_1(l)$ **do**

    $l = l + 1$; *// Find the position $l$ where the values $H_2(k)$ and $H_1(l)$ are equal*

  **end**

  $f_{21}(k) = \frac{1}{2}(k + l)$

**end**

*// Application of the contrast function $f_{12}$ to $u_1$ to compute $\hat{u}_1$*

**for** *each pixel position* $(i,j) \in \{1,\dots,N_1\} \times \{1,\dots,M_1\}$ **do**

  $\hat{u}_1(i,j) = f_{12}(u_1(i,j))$

**end**

*// Application of the contrast function $f_{21}$ to $u_1$ to compute $\hat{u}_1$*

**for** *each pixel position* $(i,j) \in \{1,\dots,N_2\} \times \{1,\dots,M_2\}$ **do**

  $\hat{u}_2(i,j) = f_{21}(u_2(i,j))$

**end**

# 3 Examples

To illustrate the efficiency and the limitations of the midway equalization, we present in this section several examples.

The method is well designed to equalize gray level images of the same scene with different exposition times (as shown in Figures 2 and 3). In that case, the cumulative histograms of the two equalized images are similar and preserve the gray level dynamics of the original images. Nevertheless, differences between images (caused, e.g. by object or camera motion , ...) can slightly modify the repartition of the gray levels in the histograms (Figure 4). Thus, the gray level of the same object in both images will not necessarily be changed in a unique gray level in the equalized images. Moreover, some quantization effects can be introduced by the method (as shown in Figure 5). These effects are generally introduced when several close pixels with similar gray levels are transformed to more distant gray levels. As a visual reduction of this effect, we use a dithering method to change randomly the gray level (as shown in Figure 5). The value of the parameter $\sigma$ of the dithering has to be chosen to reduce the quantization effects while not introducing visually disturbing noise in the images (e.g. $\sigma \approx 2$).

For color images of the same scene with different exposition times (as shown in figures 6, 7 and 8), applying the midway equalization independently over the three RGB channels of the color images usually gives good results. In those cases, the resulting images do not show color artifacts or chromatic aberrations. But, for color images from two different scenes (Figure 9), such a simple extension is not appropriate to produce suitable results. For those cases, the correct generalization of the midway equalization consists in computing Wasserstein barycenters (see [5, 6]).

One of the benefits of the midway equalization is that the method can be applied to images of different sizes. Indeed, as we use normalized cumulative histograms, the method can deal with different image sizes (as shown in Figure 10 with a reduction factor of 2 between the two images). Nevertheless, observe that too extreme differences between image sizes can increase the quantization effects presented before.


# Acknowledgments

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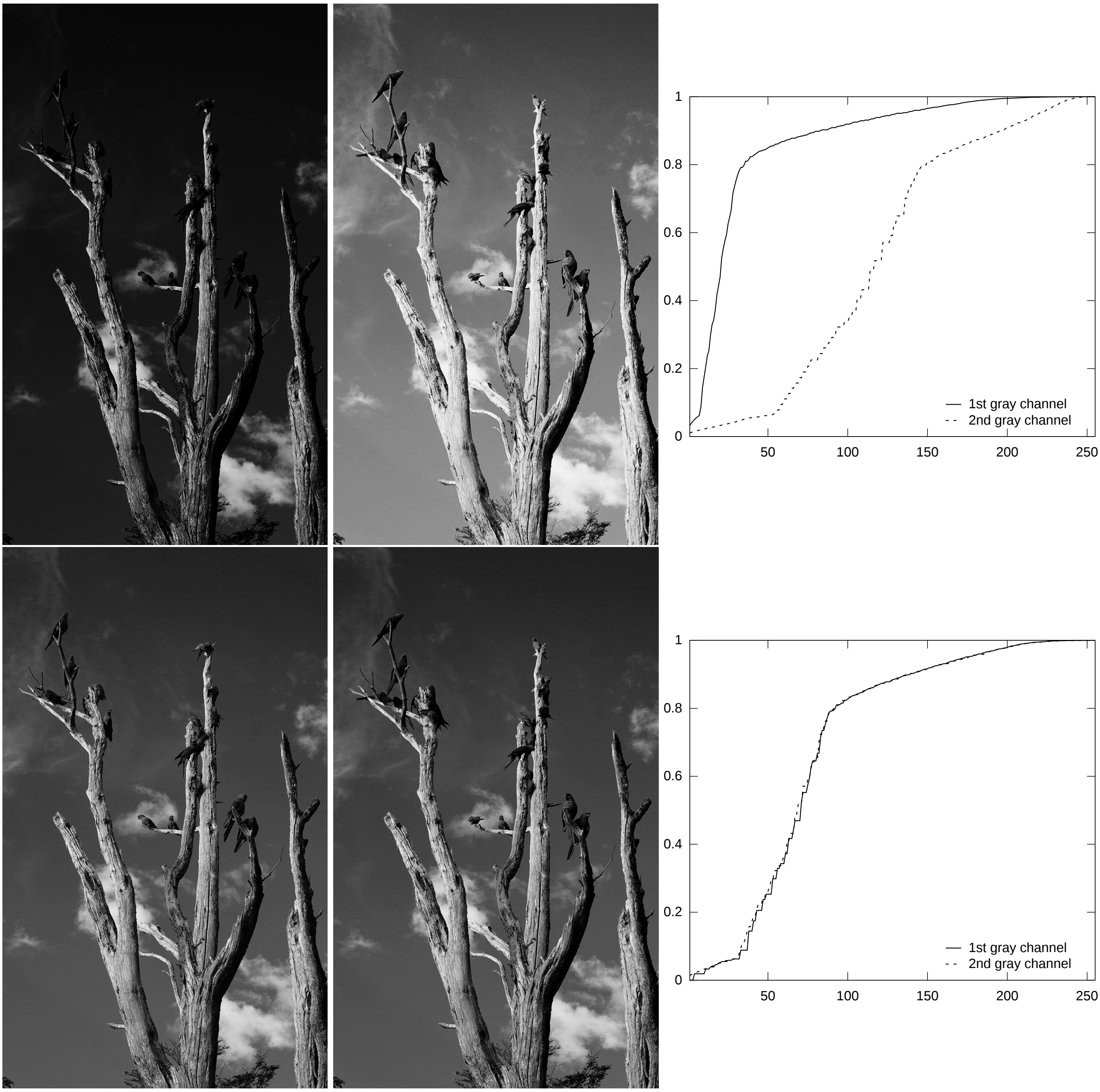


Figure 2: Midway equalization for two gray images of the same size with two different exposition times. The input images $u_1$ and $u_2$ are shown in the first row with their cumulative histograms $H_1$ and $H_2$ represented in the same figure using respectively a continuous line and a dashed line. The equalized images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row with their cumulative histograms $\hat{H}_1$ and $\hat{H}_2$ represented in the same figure using respectively a continuous line and a dashed line.

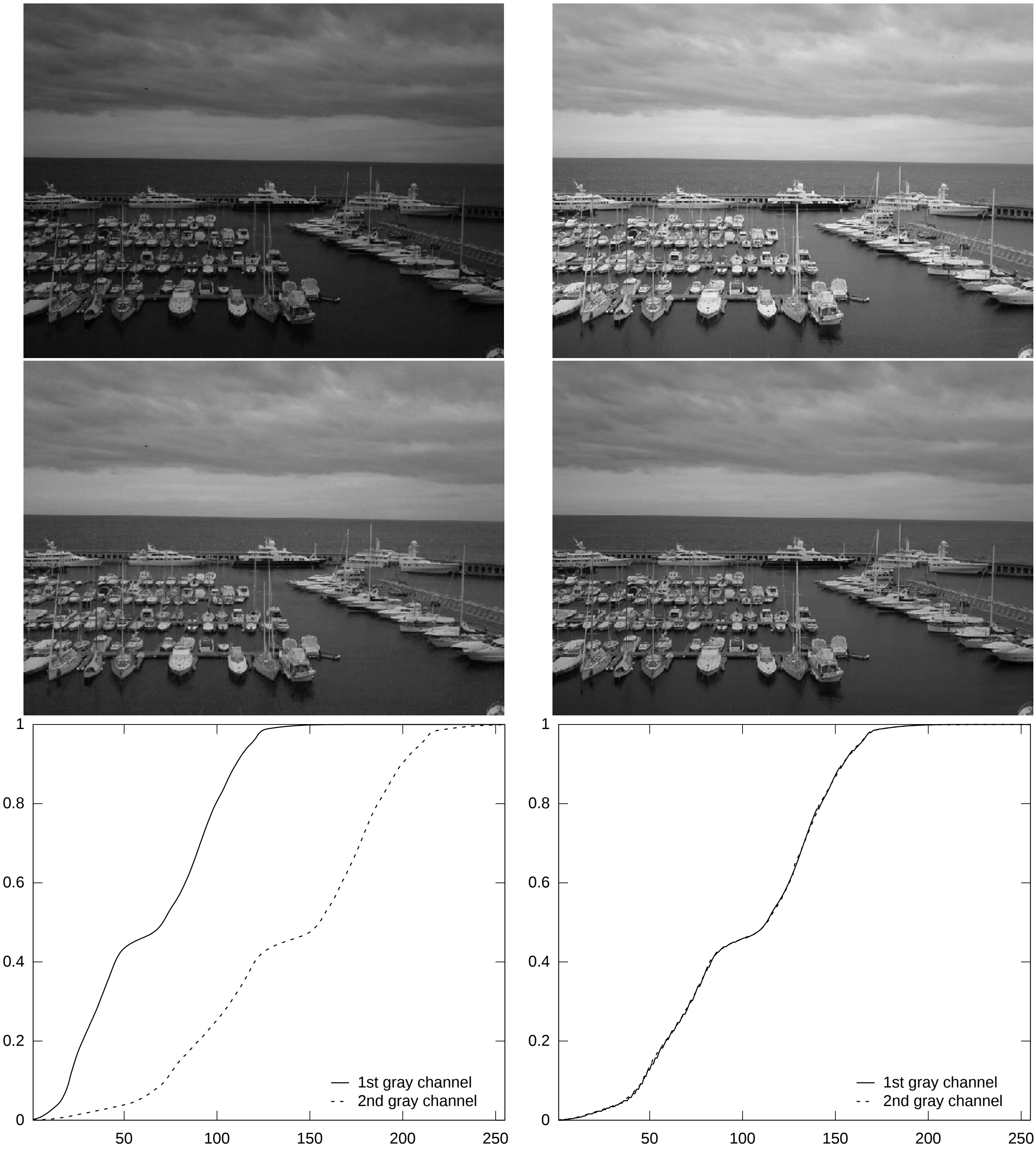


Figure 3: Midway equalization for two gray images of the same size with two different exposition times. The input images $u_1$ and $u_2$ are shown in the first row. The resulting images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row. The cumulative histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the third row.

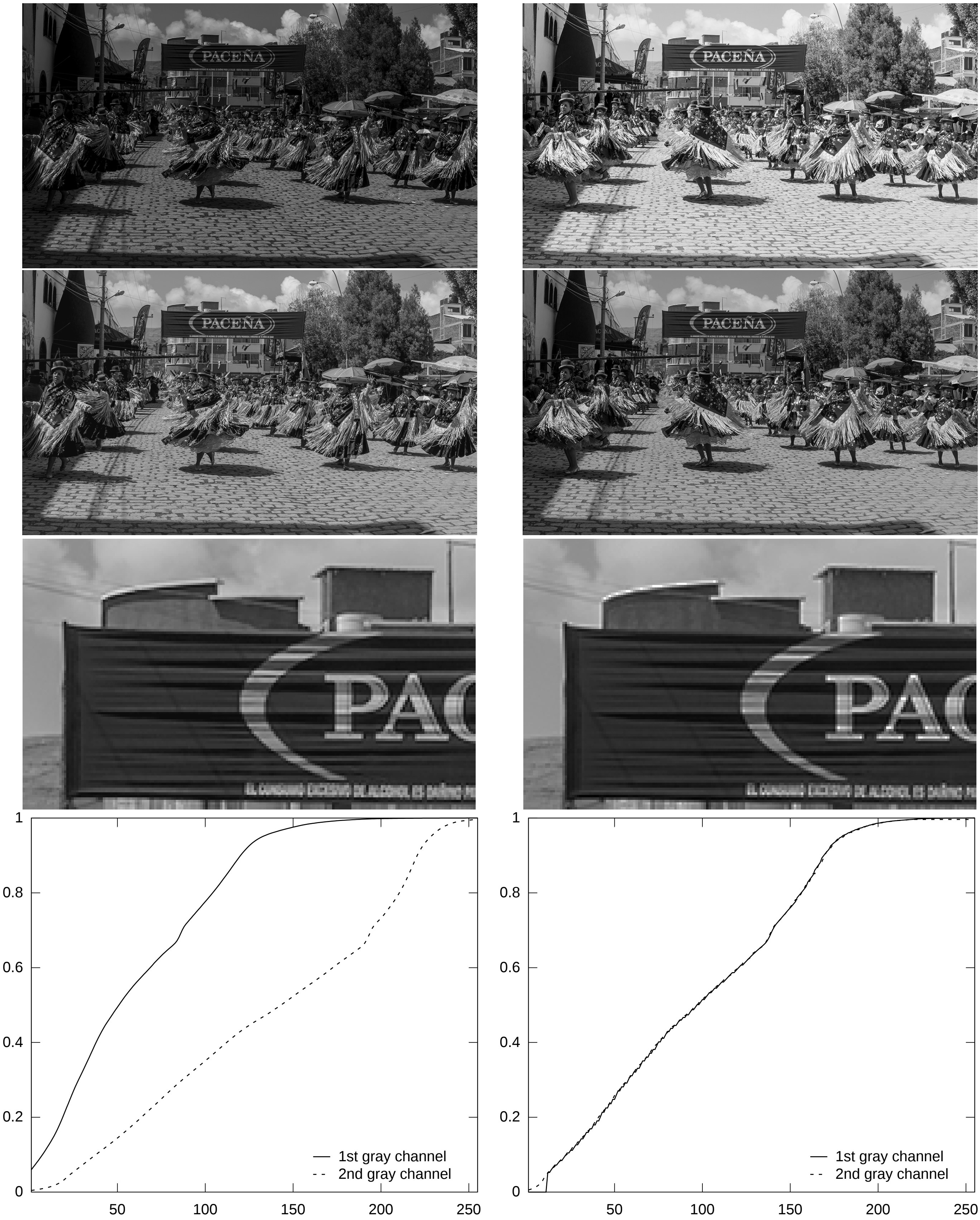


Figure 4: Midway equalization for two gray images of the same size with two different exposition times. The input images $u_1$ and $u_2$ are shown in the first row. The resulting images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row. We present in the third row a zoom of the equalized images $\hat{u}_1$ and $\hat{u}_2$ to notice the local contrast changes. The cumulative histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the fourth row.

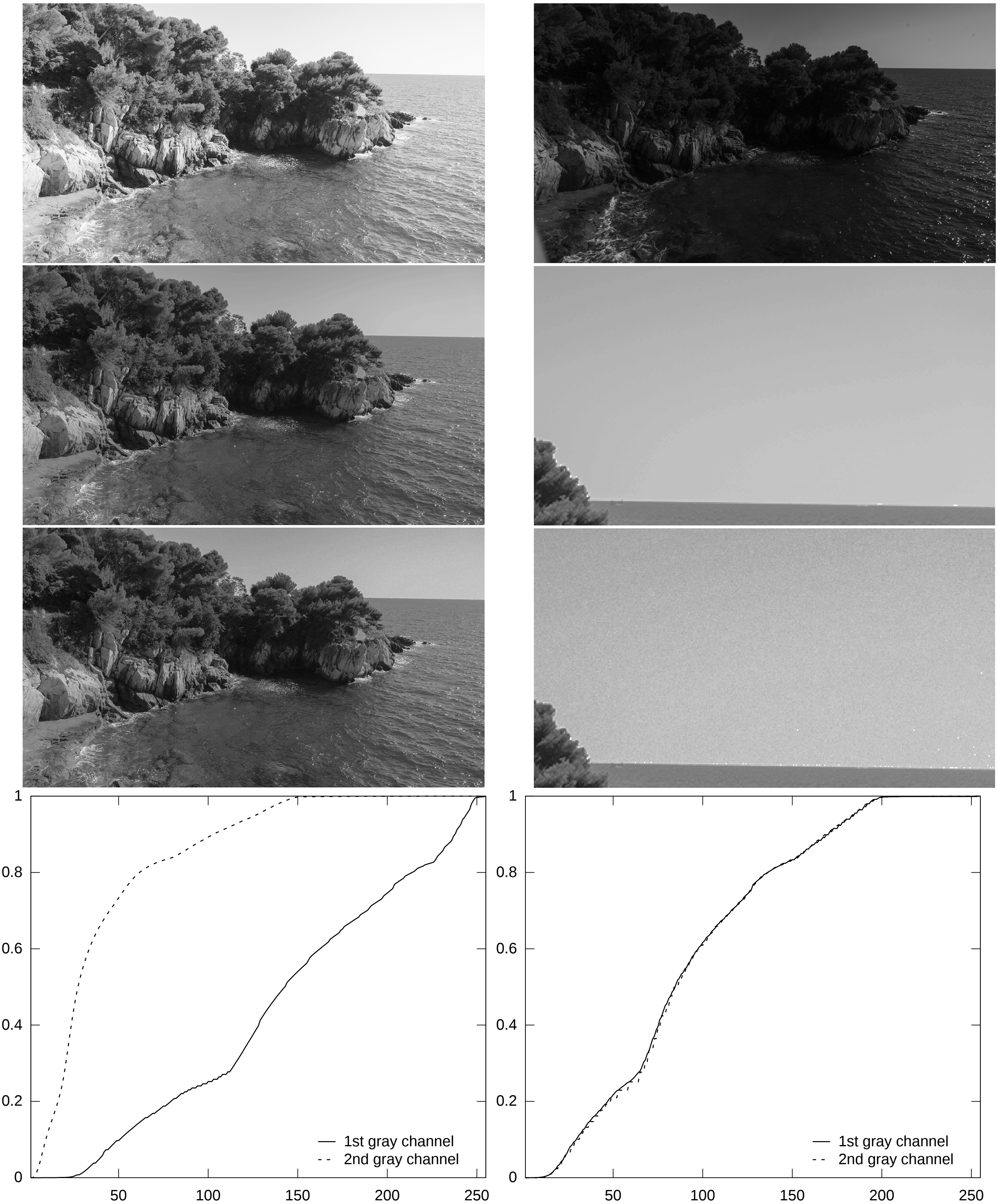


Figure 5: Midway equalization for two gray images of the same size with two different exposition times. The input images $u_1$ and $u_2$ are shown in the first row. The resulting image $\hat{u}_1$ is shown in the second row without dithering correction and in the third row with dithering correction. A detail of the images is shown to display the quantifications effects without applying a dithering correction. The cumulative histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the fourth row.

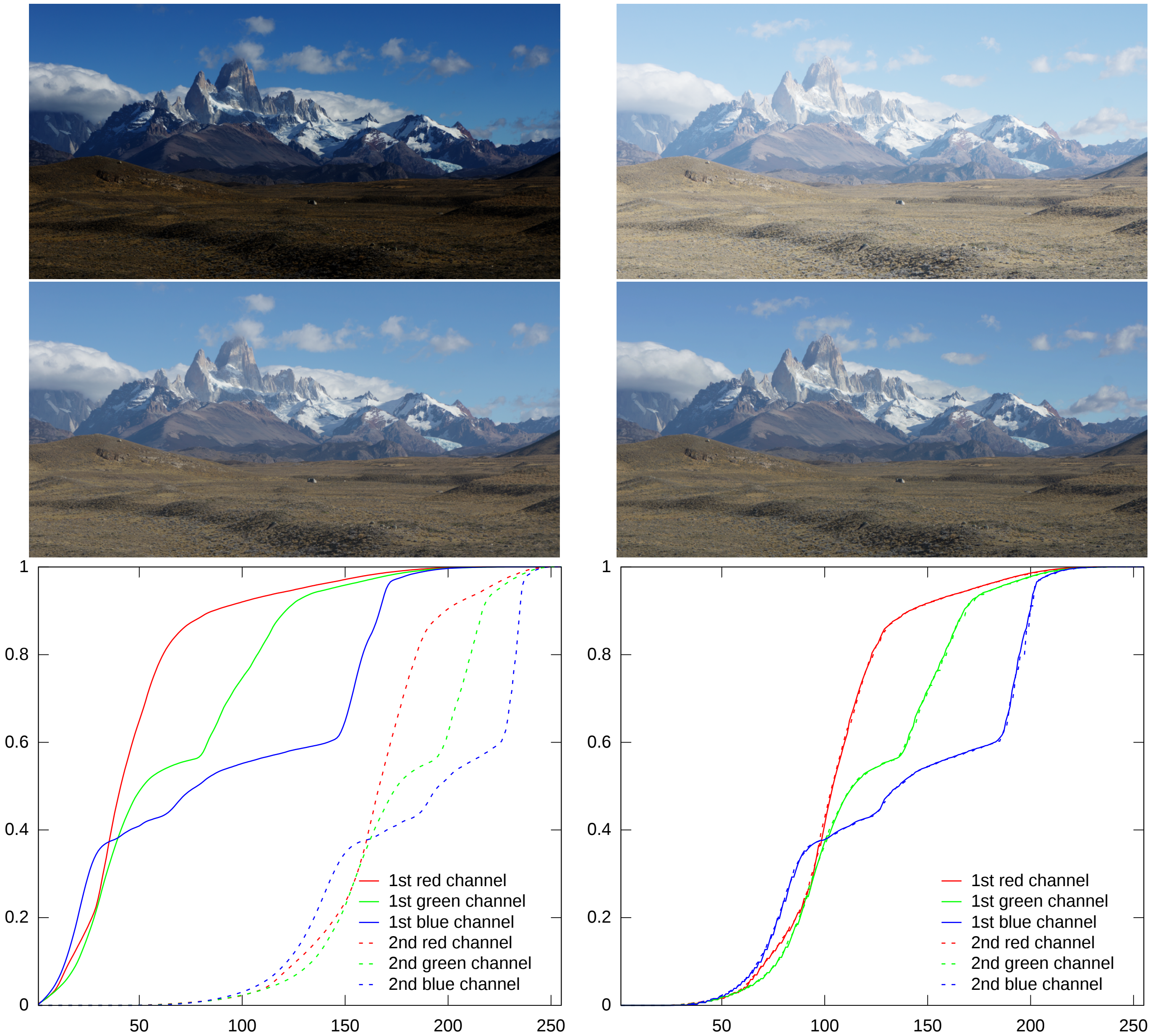


Figure 6: Midway equalization applied channel-wise for two color images of the same size taken with two different exposition times. The input images $u_1$ and $u_2$ are shown in the first row. The resulting images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row. The cumulative color histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the third row.

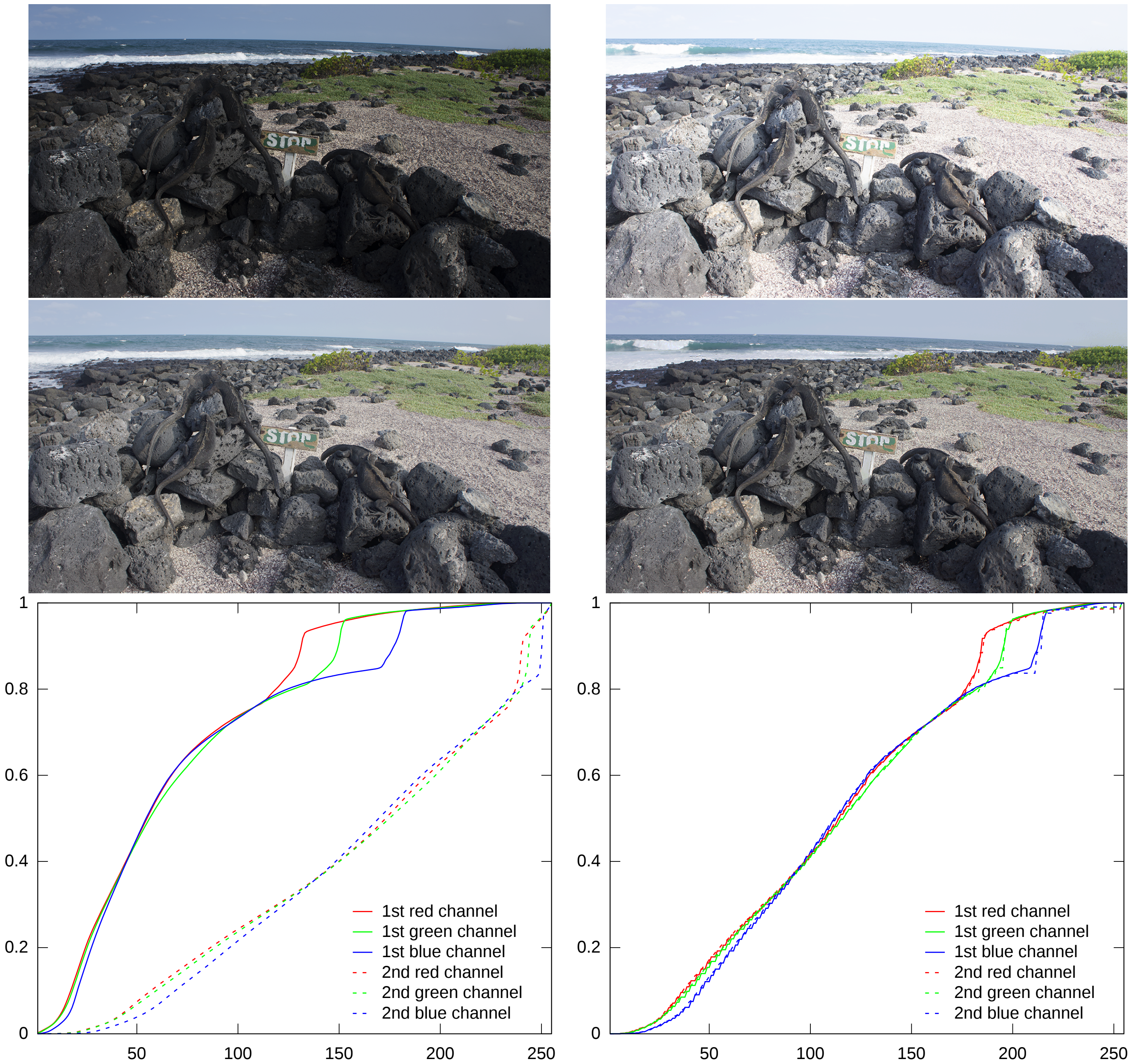


Figure 7: Midway equalization applied channel-wise for two color images of the same size taken with two different exposition times. The input images $u_1$ and $u_2$ are shown in the first row. The resulting images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row. The cumulative color histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the third row.

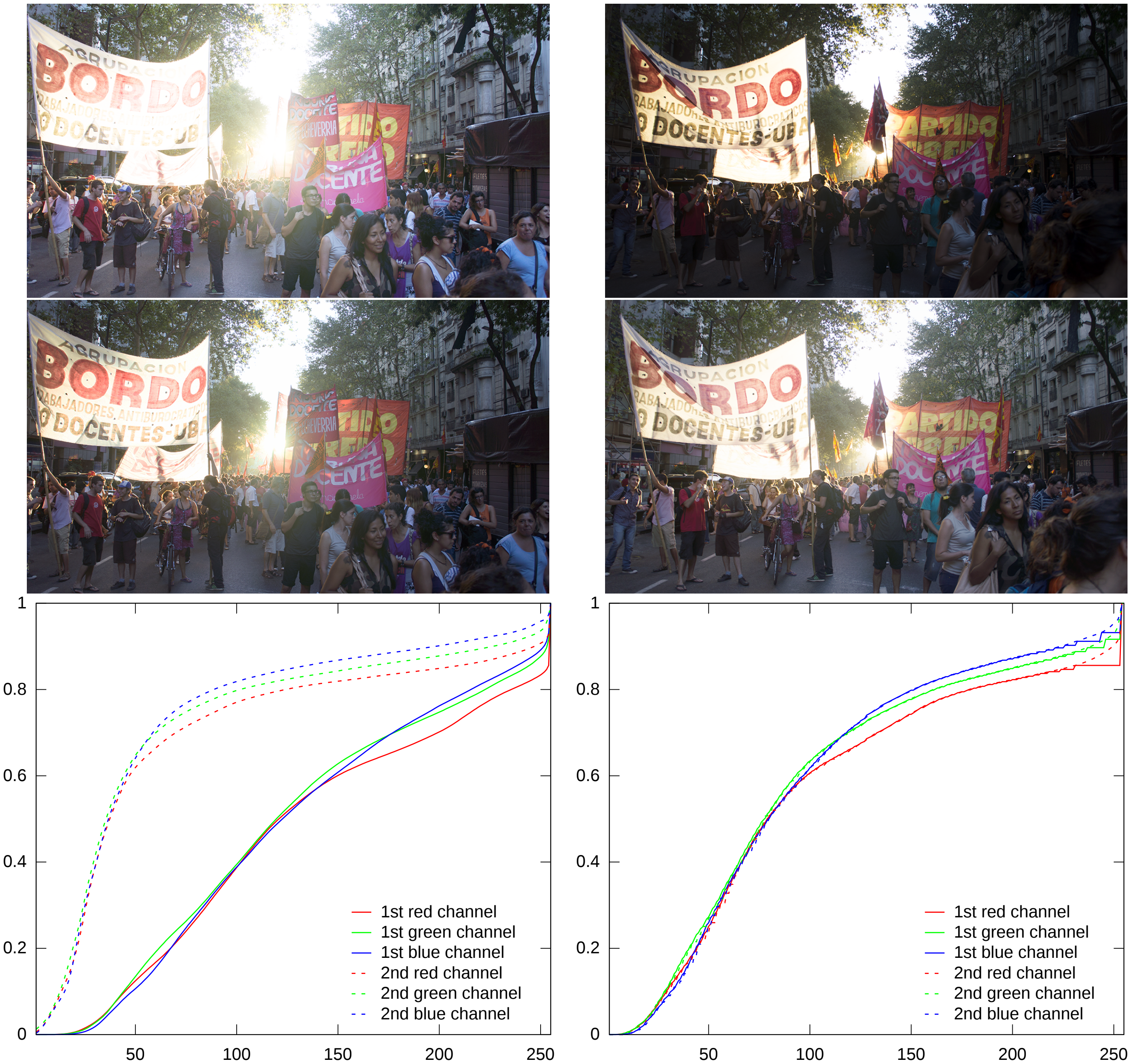


Figure 8: Midway equalization applied channel-wise for two color images of the same size taken with two different exposition times. The input images $u_1$ and $u_2$ are shown in the first row. The resulting images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row. The cumulative color histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the third row.

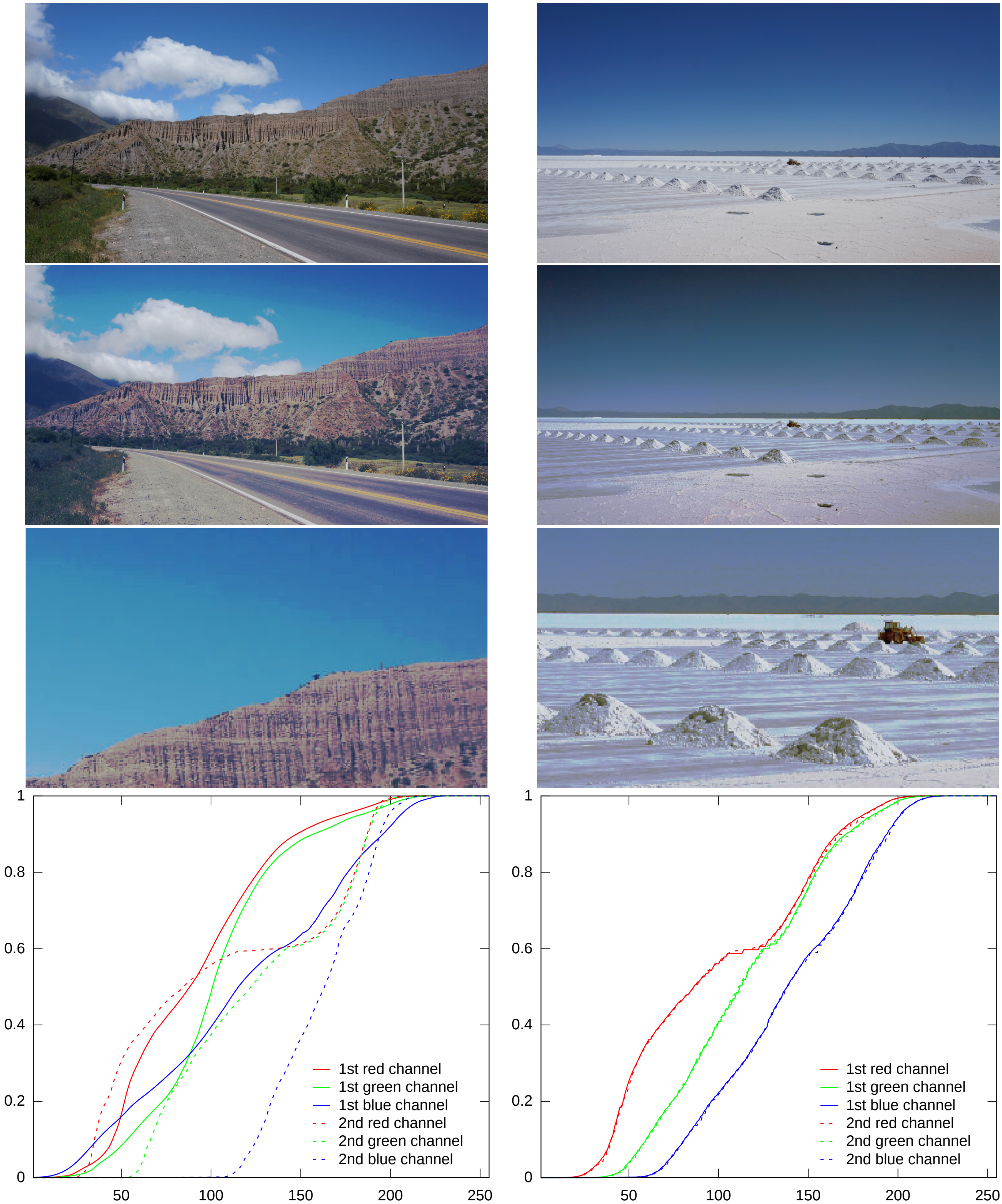


Figure 9: Midway equalization applied channel-wise for two color images taken from two different scenes. The input images $u_1$ and $u_2$ are shown in first row. The resulting images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row and a zoom of those images in the third row. The cumulative color histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the last row.

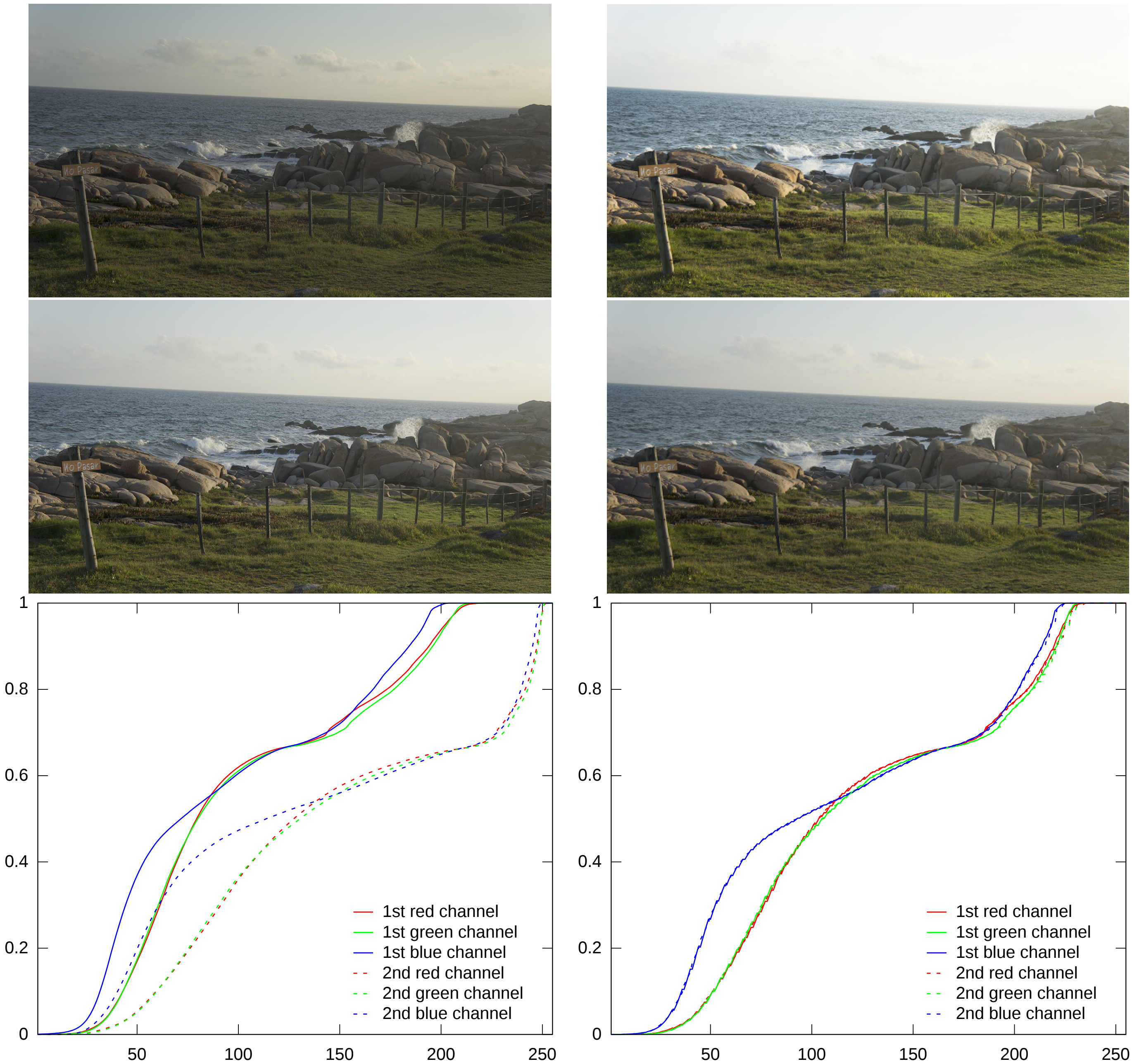


Figure 10: Midway equalization applied channel-wise of two color images of different sizes (the size of the second image is 1/4 th of the size of the first one). The input images $u_1$ and $u_2$ are shown in the first row. The resulting images $\hat{u}_1$ and $\hat{u}_2$ are shown in the second row. The cumulative histograms $H_1$ (continuous line) and $H_2$ (dashed line) of the original images and $\hat{H}_1$ (continuous line) and $\hat{H}_2$ (dashed line) of the resulting images are shown in the third row.

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